Reflection of an electromagnetic pulse incident on a nonlinear medium

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The reflection and transmission of an electromagnetic wave pulse incident on a strongly nonlinear medium are considered. An exact analytical solution is found for a rather general medium model that is characterized by two arbitrary parameters. It is shown that almost all the incident energy of some pulse forms can be reflected at large amplitudes, whereas small amplitude fields are transmitted through the medium. For other pulse forms, part of the reflected pulse can reverse its polarization due to the time dependence of the reflection coefficient. [S1063-651X(97)08212-3]

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In the past, there has been a great deal of interest [1] in investigating various nonlinear aspects of powerful laser beams which propagate through nonlinear dispersive media. The nonlinear effects considered so far include parametric instabilities, self-focusing, specular reflection, etc. Thus the emphasis was placed on the radiation-pressure-induced density fluctuations and the temporal and spatial modulations of intense radiation, as well as the self-guiding of laser beams through self-induced density channels.

In this Brief Report, we shall consider the propagation of a powerful electromagnetic wave pulse in a strongly nonlinear medium. Contrary to the traditional approach [1], the electric displacement of the medium will be modeled by a wide class of functions which are *not* described by the sum of a linear and a small nonlinear correction. Such media can be of interest as power limiters protecting for damages of strong electromagnetic pulses of natural or manmade origin.

We will calculate the reflection coefficient, and show that it can have a peculiar time dependence, thus changing the amplitude as well as the polarization of the trailing part of the reflected wave. Such anomalous reflection phenomena can also occur in nonstationary plasmas [2,3].

Let us consider a transverse, linearly polarized, electromagnetic wave propagating along the z direction in a lossless isotropic nonlinear dielectric. The displacement vector $\mathbf{D}$ is supposed to be a function of the electric field. We thus have to solve the two Maxwell equations

$$\frac{\partial E}{\partial z}=-\frac{\partial B}{\partial t} \tag{1}$$

and

$$-\frac{\partial B}{\partial z}=\frac{1}{c^2}\frac{\partial D}{\partial t}, \tag{2}$$

where $D=D(E)$, E is the x component of the electric field, B is the y component of the magnetic field, and c is the speed of light. Alternatively, we could have considered a circularly polarized wave propagating along an external magnetic field, and interpreted E as E_x+iE_y and B as B_x+iB_y, thus including electron-cyclotron waves, whistlers, etc.

Instead of looking for solutions with $E=E(z,t)$ and $B=B(z,t)$, we shall solve Eqs. (1) and (2) below by a hodograph transformation [2]. We will thus regard E and B as independent variables, and try to find solutions

$$z=z(E,B) \quad \text{and} \quad t=t(E,B). \tag{3}$$

Keeping in mind that Eq. (1) should be automatically satisfied, we then introduce a new function $\psi=\psi(E,B)$ defined from $t\equiv-\partial\psi/\partial E$ and $z\equiv\partial\psi/\partial B$. Equation (2) is accordingly replaced by

$$\frac{\partial^2\psi}{\partial E^2}-\frac{\partial D}{c^2\partial E}\frac{\partial^2\psi}{\partial B^2}=0. \tag{4}$$

Instead of ψ we now introduce the new function $F\equiv\psi\sqrt{U}$, where $U=U(E)=[1+(s_1E/E_1)+s_2E^2/E_2^2]^{-1}$. Here E_1 and E_2 are constants, $s_1=\pm1$, and $s_2=\pm1$. Furthermore, we introduce the new dimensionless variable $\varphi=\varphi(E)$ from $\varphi\equiv(1/E_c)\int_0^E U(E)dE$, where the normalizing constant E_c represents a characteristic field that will be specified below. We then obtain, from Eq. (4)

$$\frac{1}{E_c^2}\frac{\partial^2F}{\partial\varphi^2}-\frac{1}{c^2U^2}\frac{\partial D}{\partial E}\frac{\partial^2F}{\partial B^2}=p^2F, \tag{5}$$

where $p^2=(1/2U^3)(\partial^2U/\partial E^2)-(3/4U^4)(\partial U/\partial E)^2\equiv(1/4E_1^2)-s_2/E_2^2$. Our particular choice of U above has thus resulted in a *constant* value of p.

In order to be able to solve Eq. (5), we shall now limit our analysis to the (rather general) class of functions $D(E)$ for which $(\partial D/\partial E)/U^2$ is equal to a constant (which we denote by n_0^2). Furthermore, we determine the normalization constant E_c to be equal to $1/p$, and normalize B so that $b=pcB/n_0$. Equation (5) is then simplified to the Klein-Gordon equation

$$\frac{\partial^2 F}{\partial \varphi^2}-\frac{\partial^2 F}{\partial b^2}=F. \tag{6}$$

Using standard solutions of Eq. (6), we have thus been able to deduce a rather general class of solutions to the system of equations (1) and (2). Those solutions involve two characteristic medium parameters E_1 and E_2.

In order to illustrate our solution above, below we shall consider the case where the parameter $E_1=\infty$ and $s_2=-1$, i.e.,

$$U=(1-E^2/E_2^2)^{-1} \quad \text{and} \quad p=1/E_2. \tag{7}$$

It is then easy to find simple explicit expressions for D and φ, or

$$D=\frac{E_2 n_0^2}{2}\left[\operatorname{arctanh}\xi+\frac{\xi}{1-\xi^2}\right] \tag{8}$$

and

$$\varphi=\tfrac{1}{2}\ln\frac{1+\xi}{1-\xi}, \tag{9}$$

where $\xi\equiv E/E_2$ with $0\leqslant\xi<1$.

Although the specific choice (8) does not represent the general case, it can closely model practical nonlinear media such as those occuring in nonlinear geometric optics [2] as well as in plasma physics [4], where one has the possibility of varying the initial electromagnetic wave envelope on account of the harmonic and radiation pressure nonlinearities. Several examples of such media can be found in Refs. [2] and [4], where the electric field dependence of the displacement vector differs considerably from the well known Kerr and saturable nonlinearities.

As a solution of Eq. (6), we first choose

$$F=A\cosh[M\varphi-(M^2-1)^{1/2}b], \tag{10}$$

where A and $M(>1)$ are constant parameters. From Eq. (3), we then have

$$-\frac{ct}{L}=\frac{\xi}{(1-\xi^2)^{1/2}}\cosh\eta-\frac{M}{(1-\xi^2)^{1/2}}\sinh\eta \tag{11}$$

and

$$\frac{z}{L}=\frac{(M^2-1)^{1/2}(1-\xi^2)^{1/2}}{n_0}\sinh\eta, \tag{12}$$

where $\eta\equiv M\varphi-(M^2-1)^{1/2}b$, and where $L\equiv-pcA$ is a characteristic scale length defined by the nonlinear properties of the medium.

Using Eqs. (11) and (12) to express $\sinh\eta$ and $\cosh\eta$ as functions of z,t, and ξ, and using the relation $\cosh^2\eta-\sinh^2\eta=1$, we then obtain the equation

$$\left[\frac{z}{L}-\frac{vt}{L}(1-\xi^2)\right]^2-\left(\frac{z\xi}{LM}\right)^2=\xi^2(1-\xi^2)\frac{v^2}{c^2}, \tag{13}$$

where $v\equiv c(1-1/M^2)^{1/2}/n_0$.

Using Eq. (13) we have thus found ξ (the normalized electric field) as a function of z and t. Inserting ξ into Eq. (12), and using Eq. (9), we can then directly also find b (the normalized magnetic field).

Let us finally consider a plane wave incident from vacuum ($z<0$) on a nonlinear medium ($z\geqslant 0$). From $\partial\psi/\partial B|_{z=0}=0$ we have the boundary condition $M\varphi_0-(M^2-1)^{1/2}b_0=0$. By means of Eq. (9), or $2\varphi_0=\ln[(1+\xi_0)/(1-\xi_0)]$, we rewrite that relation as

$$\frac{E_0}{E_2}=\tanh[cB_0(1-1/M^2)^{1/2}/n_0E_2], \tag{14}$$

where $E_0=E(t)|_{z=0}, B_0=B(t)|_{z=0}, \varphi_0=\varphi|_{z=0}$, and $b_0=b|_{z=0}$.

The incident wave is transmitted and reflected in a standard manner, where the continuity of the tangential components of the total electric and magnetic fields has been taken into account. We thus find the reflection coefficient $R(\equiv|r|^2)$ from the relation

$$\frac{1+r}{1-r}=\frac{E_0}{cB_0}, \tag{15}$$

i.e.,

$$r=\frac{(v/c)-G(\xi_0)}{(v/c)+G(\xi_0)}, \tag{16}$$

where $G(\xi_0)=[\operatorname{arctanh}(\xi_0)]/\xi_0$.

From Eq. (13) we easily find that $\xi_0=(1+L^2/c^2t^2)^{-1/2}$. We also note that Eq. (16) changes from $r\approx(v-c)/(v+c)<1$ when $\xi_0\approx 0$ to $r\approx-1$ when $\xi_0\approx 1$, which indicates a strong self-screening effect.

As another solution of Eq. (6) we may choose $F=A\cos[M\varphi-(1+M^2)^{1/2}b]$, where A and M are constant parameters. The calculations are analogous to those above. Instead of Eq. (13) we here obtain the same terms, but with a different sign on the second term on the left-hand side. Formula (16) is also essentially the same, with v/c now replaced by $(1+1/M^2)^{1/2}/n_0$, and G by $[\arctan(\xi_0)]/\xi_0$.

To summarize, we found that the front of the field pulse propagates with the velocity $v=(c/n_0)|1\mp 1/M^2|^{-1}$, which is equal to the speed of light inside the medium (c/n_0) times a factor which depends on the pulse parameter M that is related to the front steepness. Furthermore, we note that the electric and magnetic field envelopes of the transmitted pulse are *not* proportional to each other, and that our first solution (10) shows a strong self-screening effect ($r\rightarrow-1$ when

$\xi_0 \rightarrow 1$) which means that our nonlinear medium can act as a power limiter. With our second solution of Eq. (6) it turns out that the coefficient r changes sign when $(\arctan\xi_0)/\xi_0 = v/c$, i.e., the nonlinear reflection changes the polarization of the trailing part of the reflected pulse. Thus, depending on the chosen solution, we can have either self-screening without change of polarization, or polarization reversal but then without the self-screening effect.

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